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Managing Interest Rate Risk by Managing Duration

In a perfect world, interest rates would not fluctuate and the value of the bonds in your portfolio would not change over time.

Unfortunately, we live in a world where interest rates move up and down...sometimes dramatically...over the course of a bond's term. This movement can have a meaningful impact on your portfolio, if not taken into account in the day-to-day management of your fixed income assets.

One of the ways we help to insulate your portfolio from negative interest rate moves and to take advantage of those that are positive, is to carefully monitor its duration. In what follows, we will examine the concept of duration and its applicability as an interest rate risk management tool.

Duration as a risk measure:

There is an inverse relationship between the price of a bond and its yield. The yield on a bond is the discount rate that makes the present value of the bond's cash flows equal to its price. Thus, as the bond's price declines, its yield increases, a direct reflection of the discount between the bond's par value and the price investors are willing to pay for it.

In a rising interest rate environment investors demand greater yields, which can only be provided by lowering the asking price for the bond. Conversely, in a decreasing interest rate environment, the bond's fixed rate of return may make it very attractive to investors, causing its price to rise (a reflection of increased demand) and its yield to decline.

Obviously, this kind of price variability can play havoc on the value of a fixed income portfolio at any given point in time, unless the portfolio is carefully monitored and positioned to defend against or take advantage of changes in the interest rate environment.

Duration, which measures the price sensitivity of bonds to a variable yield environment, enables us to do just that.

There are three primary duration calculations: Macaulay Duration, Modified Duration and Effective Duration.

Macaulay Duration:

In 1938, Frederick Macaulay introduced the concept now known as "Duration". Named after its founder, Macaulay Duration is a calculation of the approximate sensitivity of a bond's price to changes in interest rates.

Macaulay Duration

$$= \frac{\sum (\text{Number of Years} \times \text{Present Value of Cash Flows})}{\text{Price of the bond}}$$

Where Σ = Sum of

In general, Macaulay Duration is used to determine the point in time, measured in years, at which half of a bond's total cash flows will be received. However, to make it more usable as a mathematical calculation of risk, Macaulay Duration has been modified slightly, giving rise to a more modern interpretation known as "Modified Duration".

Modified Duration:

Modified Duration equals Macaulay Duration divided by one plus the yield to maturity. Since Modified Duration and Macaulay Duration essentially measure the same thing (i.e., sensitivity of a bond's price to changes in yields or interest rates), one measure is not strictly preferable to the other. However, for estimating price changes, Modified Duration is easier to use, as can be seen in the following equation.

Approximate Percentage Change in Price

$$= - (\text{Modified Duration} \times \text{Yield Change})$$

As determined by the Modified Duration method, duration is the approximate percentage change in a given bond's price for each 100 basis point (bp) shift in the yield curve. For example, if a bond's duration is 4.0, this means a 1.0% (100 basis point) decrease in interest rates will result in a price increase of approximately 4.0%. By the same token, a 50 basis point increase in yields will result in a price decline of approximately 2.0%.

Effective Duration:

Modified Duration is a good way to measure bonds that have fixed cash flows and maturity dates, but it can't accurately measure the price/interest rate sensitivity of bonds that have embedded options, such as mortgage backed securities and callable corporate bonds.

Mortgage backed securities are complicated by the fact that homeowners have the *option* of refinancing their mortgages when interest rates drop. When this option is exercised, investors in mortgage backed securities get their principal back much earlier than expected and are faced with having to reinvest the proceeds in lower yielding instruments.

Similarly, with callable corporate bonds, issuers have the right to call, or redeem, the bond prior to maturity. As with the homeowner, corporations tend to call these bonds when interest rates fall, in order to reduce the borrowing cost of the loan. Again, the investor must reinvest in lower yielding instruments.

We have to quantify and adjust for redemption-related risk when analyzing the characteristics of a bond with embedded options. Modified Duration does not take into account this risk of optionality, if you will, because it assumes the yield to maturity is not affected by changes in interest rates.

The only duration formula that can measure the risk of bonds that have embedded options is called “Effective Duration,” or “Option Adjusted Duration”. Effective Duration adjusts the riskiness of bonds by taking into account the relative sensitivity of bonds with different coupon rates and terms to prevailing interest rates. It can gauge the likelihood of the call being exercised, or a mortgage being refinanced, and thereby help to measure the risk represented by these securities in various yield, or interest rate, environments. We would note that Effective Duration should be equal to or close to Modified Duration for bulleted bonds, (i.e. those without option risk)

Now, let’s review how these duration formulas can be applied to different types of bonds.

Example:

Here, we will calculate Macaulay Duration, Modified Duration and Effective Duration for a bond with the following characteristics:

Coupon: 6.0 % (annually)
Number of years to maturity: 3 years
Price: \$97.11
Yield to maturity (YTM): 7.1% or 0.071

Years		YTM = 0.071	Discount Factor	Coupon	Cash Flow		
N	(N+ 1)	(1+ YTM)	Df	C	PV = Df x C	N (PV)	(N) (N+1) (PV)
1	2	1.071	0.93	\$6.00	\$5.60	5.60	11.20
2	3	1.071	0.87	\$6.00	\$5.23	10.46	31.38
3	4	1.071	0.81	\$6.00	\$4.88	14.65	58.60
3	4	1.071	0.81	\$100.00	\$81.40	244.20	976.81
				Total	\$97.11	274.92	1078.01

Macaulay Duration

$$= \frac{\sum (\text{Number of Years} \times \text{Present Value of Cash Flows})}{\text{Price of the bond}}$$
Where Σ = Sum of

With this bond, **Macaulay Duration** is equal to

$$\begin{aligned} &= 274.92 \div 97.11 \\ &= 2.83 \text{ years} \end{aligned}$$

The **Modified Duration** for this bond would be calculated by using the following formula:

$$\text{Modified Duration} = \frac{\text{Macaulay Duration}}{(1 + y)}$$

Where y = yield to maturity or YTM

Therefore, the **Modified Duration** for this bond is equal to:

$$\begin{aligned} &= 2.83 \div 1.071 \\ &= 2.64 \end{aligned}$$

For the sake of simplicity, we will calculate the **Effective Duration** for the above bond with a few assumptions. We will calculate the Effective Duration from a hypothetical change in bond prices, as interest rates move up or down by 2%. If interest rates move down, the issuer will exercise the call option, which is the additional risk we will quantify. The call price is assumed to be 100.

The details of this bond are as follows:

Coupon:	6% annual payment
Number of years to maturity:	3 years
Price:	\$97.11
Yield to maturity (YTM):	7.1% or 0.071
Call Price:	\$100
Interest Rate Increase:	2% or (+0.02)
Interest Rate Decrease:	2% or (-0.02)
Modified Duration:	2.64

For a **2.0% (or .02) increase in interest rates**, the percentage change, or decrease, in the bond's price would be as follows:

$$\begin{aligned} \text{Approximate Percentage change in price} &= -2.64 \times (+0.02) \\ &= -0.0528 \\ &= -5.28\% \text{ decrease in price} \\ \text{Adjusted Price} &= \text{Initial Price} \times \text{Percent decrease in Price} \\ \text{Adjusted Price} &= \$97.11 \times (-5.28\%) \end{aligned}$$

For a **2% (or -0.02) decrease in interest rates**, the percentage change, or increase, in the bond's price would be as follows:

Approximate Percentage change in price	=	$-2.64 \times (-0.02)$
	=	0.0528
	=	5.28% Increase in price
Adjusted Price	=	Initial Price \times Percent increase in Price
Adjusted Price	=	$\$97.11 \times (5.28\%)$

Therefore, if interest rates increase by 2.0% (or 0.02), the bond's price would decline by approximately 5.28%, to \$91.98. Similarly, it would at first appear that when interest rates decrease by 2.0%, the bond's price would increase by 5.28%, to \$102.24. However, that would not be the case.

With a reduction in interest rates, the call option would kick in and the bond would most likely be called at \$100. Subsequently, as interest rates decrease, the bond would trade very close to or equal to its call price. As such, the bond price would be close to or equal to \$100 and not \$102.24.

Taking this into account, the Effective Duration of this bond in a declining rate environment would be calculated as follows:

Approximate Percentage change in price due to a decrease in interest rates by 2% = 100
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A simple formula for calculating Effective Duration in this situation is:

Effective Duration
= $\frac{V_- - V_+}{2 \times V_0 \times (\Delta y)}$

Where:

- | | | |
|------------|---|--------------|
| V_0 | = Initial Price of the security | = \$97.11 |
| V_- | = Estimated value of security if the yield is decreased by Δy | = \$100.00 |
| V_+ | = Estimated value of security if the yield is increased by Δy | = \$91.98 |
| Δy | = Change in the yield of a security | = 2% or 0.02 |

Therefore, using these numbers, we arrive at the following Effective Duration:

$$\begin{array}{r} \text{Effective Duration} \\ = \frac{100.00 - 91.98}{2 \times (97.11) \times (0.02)} \\ = 2.06 \end{array}$$

The Effective Duration number of 2.06 is an accurate approximation of the sensitivity of the cash flows of a bond with an embedded option, to changes in yields or interest rates.

If we used the price of \$102.24, instead of \$100, in calculating this bond's Effective Duration in a declining interest rate environment, the Effective Duration would be equal to Modified Duration. Let's find out why...

$$\begin{array}{r} \text{Effective Duration} \\ = \frac{102.24 - 91.98}{2 \times (97.11) \times (0.02)} \\ = 2.64 \end{array}$$

Under this circumstance, the \$2.24 premium fully compensates the investor for the option risk and is a good representation of the cost of the option. And, as we mentioned earlier, Effective Duration will equal or be very close to Modified Duration for bulleted bonds (i.e. those without option risk). Now that the optionality is fully priced into the value of the bond, the Effective Duration and Modified Duration calculations are the same.

In conclusion, duration can be interpreted as the approximate percentage change in price for each 1.0% shift in the yield curve, and as such is an important measure of a bond's sensitivity to changing interest rates. By actively managing this risk factor, we can control your portfolio's exposure to external market forces. By lengthening the duration of your portfolio, for example, we can enhance your overall returns when interest rates fall. And, by shortening your portfolio's duration, we can mitigate potential underperformance when interest rates rise.

Please let us know if you have any questions concerning the duration of your portfolio, or if you would like a more detailed examination of the concept of duration and its applicability in the process of managing fixed income interest rate risk.

Vanderbilt Research Team